TRINITY COLLEGE

IN NOMINE DOMINI

Semester Two Examination, 2021

Question/Answer booklet

MATHEMATICS METHODS UNITS 3&4

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

WA student number: In figures   ☐☐ ☐☐☐ ☐☐☐

In words   _____

_____

Your name   _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable): ☐

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
				Total	100

Instructions to candidates

1.   The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2.   Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3.   You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4.   Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5.   It is recommended that you do not use pencil, except in diagrams.

6.   Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7.   The Formula sheet is not to be handed in with your Question/Answer booklet.

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Section Two: Calculator-assumed   **65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9   **(8 marks)**

An online employment survey on a public internet forum attracted 75 responses from mine workers, of whom 30 said that they were employed on a temporary contract.

(a)   Use the survey data to construct a 90% confidence interval for the population proportion of mine workers employed on a temporary contract.   (3 marks)

(b)   Assuming the survey was reliable, determine the sample size required to conduct a follow-up survey so that a 95% confidence interval for the population proportion of mine workers employed on a temporary contract will have a margin of error close to 0.05.   (3 marks)

(c)   Identify and explain a possible source of bias that may arise from this type of survey.
                                                                                    (2 marks)

Question 10 (8 marks)

A capacitor in a circuit starts to discharge. The voltage V across the capacitor after t milliseconds is changing at a rate given by

$$\frac{dV}{dt} = \frac{-162}{(2t+3)^2}, \qquad t \geq 0.$$

(a) Calculate the initial rate of change of voltage. (1 mark)

(b) Determine the change in voltage during the third millisecond. (3 marks)

(c) Given that the initial voltage across the capacitor was 26 volts, determine the time for the voltage to fall to 1 volt. (4 marks)

See next page

Question 11   **(7 marks)**

A company packages tea in tins marked with a net weight of 375 g. The weight of tea in the tins is normally distributed with a mean of 384.5 g and a standard deviation of 4.8 g.

(a)   Determine the proportion of tins that contain more than the marked weight.   (2 marks)

(b)   What proportion of tins containing more than the marked weight contain more than 390 g of tea?   (2 marks)

(c)   The company has decided that no more than 1 in 500 tins should contain less than the marked weight of tea. To achieve this, they will pack more tea in each tin and hence increase the mean of the distribution whilst maintaining the existing standard deviation. Determine the minimum increase in the mean required.   (3 marks)

Question 12   **(8 marks)**

A factory makes identical plastic key fobs in four different colours. 15% are red, 20% are green, 25% are blue and the remainder orange. The key fobs are randomly packed into boxes of 120.

Quality control at the factory randomly sample several boxes from the production line daily and record, amongst other things, the proportion of green key fobs in each box.

(a)   Describe the continuous probability distribution that the sample proportion of green key fobs will approximate over time, including any parameters.   (3 marks)

(b)   Use a similar continuous probability distribution to that in part (a) to calculate an approximation for the probability that the proportion of orange key fobs in a randomly chosen box is at least 35%.   (3 marks)

(c)   Briefly explain why the distribution in part (a) is an approximation and state the key factor that determines the closeness of the approximation.   (2 marks)

Question 13   **(7 marks)**

A small body starts from rest at point A and moves in a straight line until it reaches point B, where it is again stationary.

The acceleration of the body t seconds after leaving A is a m/s^2, where $a = 0.018t - 0.0003t^2$.

Determine

(a)   the time taken for the body to travel from A to B.   (3 marks)

(b)   the distance from A to B.   (2 marks)

(c)   the maximum velocity of the body between A and B.   (2 marks)

See next page

Question 14   **(8 marks)**

The level of atmospheric carbon dioxide C in parts per million was measured by scientists at an Arctic base and was observed to increase from 322.9 ppm on 1 January 1967, to 335.4 ppm by 1 January 1976.

The level can be modelled by equation $C = C_0 e^{kt}$, where t is the number of years from the start of the year 1960.

(a)   Determine an expression for the constant k in the form $a\ln(b)$ and hence show that its value is approximately 0.00422.   (3 marks)

(b)   Determine the value of the constant C_0.   (2 marks)

(c)   Calculate the level of atmospheric carbon dioxide at the start of the year 1995.   (1 mark)

(d)   Determine the rate at which the level of atmospheric carbon dioxide was increasing at the start of the year 1995.   (2 marks)

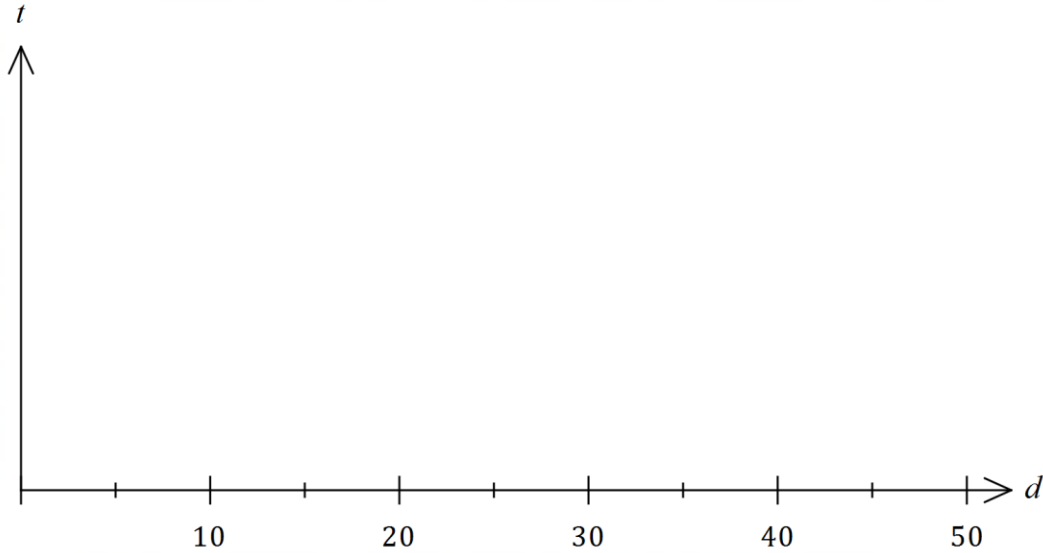**Question 15**   **(7 marks)**

The time, t seconds, for a trained rat to pick a bead out of a container and drop it into a small hole, when the distance of the bead container from the hole was d cm, can be modelled by the relationship $t = 0.6 + 0.55 \ln(d)$ for $d \geq 1$.

(a)   Sketch the graph of t as a function of d for $1 \leq d \leq 50$ cm.   (3 marks)

(b)   Determine the extra time taken by the rat to move a bead when the distance of the bead container from the hole increases from 20 cm to 60 cm.   (1 mark)

(c)   Use the relationship to show that if the distance of the bead container from the hole increases from x cm to $3x$ cm, the change in time is constant.   (3 marks)

See next page

Question 16   **(8 marks)**

A person drives to work n times each month and on any one journey, the probability that they arrive late for work is p.

(a)   When $n = 18$ and $p = 0.2$ determine the probability that

   (i)   they are never late for work in a month.   (2 marks)

   (ii)   they are late for work at least twice in a month.   (1 mark)

   (iii)   they are late for work at least once in each of six consecutive months.   (2 marks)

(b)   Determine n and p when the mean and variance of the number of times the person is late for work each month is 3.6 and 3.06 respectively.   (3 marks)

Question 17   **(8 marks)**

The local newspaper in a large city claimed that over 60% of the city's population trusted them. To check this claim, a research group took a random sample of 975 people in the city and found that 546 of them trusted the newspaper.

(a)   Construct a 95% confidence interval for the proportion of all people in the city who trust the newspaper and hence comment on the validity of the newspaper's claim.   (4 marks)

(b)   The research group carried out the same sampling task in a different city, from which the 99% confidence interval $(0.594, 0.708)$ was constructed. Determine the number of people in this sample who trusted their local newspaper.   (4 marks)

Question 18 **(6 marks)**

A player throws a regular tetrahedral die whose faces are numbered $1, 2, 3$ and 4. If the player throws a two, the die is thrown a second time, and in this case the score is the sum of 2 and the second number; otherwise, the score is the number obtained. The player has no more than two throws. Let X be the random variable denoting the player's score.

(a)   Write down the probability distribution of X.   (3 marks)

(b)   Determine the mean and standard deviation of X.   (2 marks)

(c)   Determine $P(X = 1 \mid X \leq E(X))$.   (1 mark)

See next page

Question 19   **(8 marks)**

An electronic device is powered by a AAA battery that will always last for a minimum of 12 hours. The random variable T is the number of hours exceeding 12 for which the device will continue to operate, and it has a probability density function $f(t)$ shown below:

$$f(t) = \begin{cases} \dfrac{kt}{4} & 0 \le t \le 4 \\ k & 4 < t \le 5 \\ 0 & \text{otherwise} \end{cases}$$

(a)   Determine the value of the constant k.   (3 marks)

(b)   Calculate the mean of T.   (2 marks)

(c)   Given that $P(T \ge a) = 0.865$, determine the value of the constant a. (3 marks)

Question 20   **(8 marks)**

In the diagram, $ABCDE$ is part of a plan of the interior of an office, where walls BC and CD have lengths 5 and 2 m respectively. An electrician must run a cable from point P on wall AB in a straight line to point Q on wall DE so that it just touches corner C. The angle the cable PQ makes with wall AB is θ.
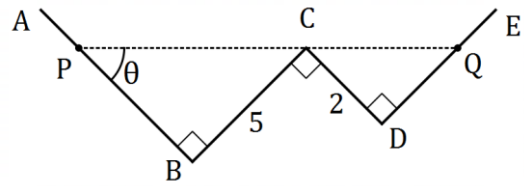
(a)   When $\theta = 32°$, show that the length L of the cable is approximately 11.8 m.   (2 marks)

(b)   Use a calculus method to determine the optimum angle that minimises the length L of the cable, state what this minimum length is and justify that it is a minimum.   (6 marks)
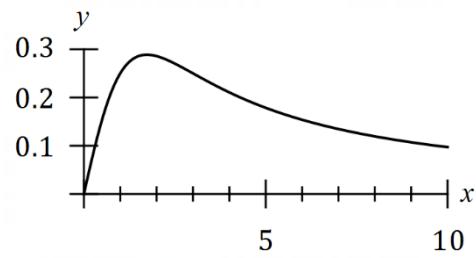
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Question 21   **(7 marks)**

The graph of $y = f(x)$ is shown, where

$$f(x) = \frac{x}{3 + x^2}, \qquad x \geq 0.$$

$f(x)$ is concave down for $0 < x < 3$.

(a)   Determine the area bounded by the graph of $f(x)$ and the line $y = \frac{x}{12}$.   (3 marks)

The line $y = mx$ and the graph of $f(x)$ enclose a finite region R.

(b)   Determine the values of the slope m for which R exists.   (2 marks)

(c)   Determine the area of R in terms of m.   (2 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

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